METHODS OF IDENTIFYING THE PARAMETERS OF THERMOPHYSICAL OBJECTS AND THEIR SCHEMATIC REALIZATION

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Methods of identifying the thermal diffusivity and thermal activity without loss of integrity and with introduction into the experimental object are considered. Identification schemes based on electrical models with tunable parameters are presented.

Systemic and structural analysis of processes of nonsteady heat transfer [1] forms the theoretical basis of various identification methods of constructing measuring and information systems and, in particular, identification systems for the parameters of thermophysical objects and thermal perturbations. According to [1], the space–time formation of a temperature field and heat fluxes at the boundary of a body and inside it is shown using a structural scheme consisting of a set of elements (blocks) with definite interrelations. Elements of the structural scheme are described by system operators (transfer functions) determining the rule governing the transformation of input perturbations into the output function; the inputs and outputs of the blocks are observable, and have clear physical significance. A correctly formulated structural scheme is essentially an informative mathematical model of the given process and provides the basis for the construction of various identification schemes.

The principles of construction of schemes for identifying the thermal-activity coefficients b and thermal diffusivity a are considered for the example of a structural scheme of heat transfer in a semiinfinite (in thermal terms) body, at the surface of which there acts a circular heat source of specific power $q(\tau)$ (Fig. 1a).

Heat transfer in the body is described by a system of differential equations

$$\frac{\partial^2 T_1(r, x, \tau)}{\partial r^2} + \frac{1}{r}\frac{\partial T_1(r, x, \tau)}{\partial r} + \frac{\partial^2 T_1(r, x, \tau)}{\partial x^2} = \frac{1}{a}\frac{\partial T_1(r, x, \tau)}{\partial \tau};$$

$$\frac{\partial^2 T_2(r, x, \tau)}{\partial r^2} + \frac{1}{r}\frac{\partial T_2(r, x, \tau)}{\partial r} + \frac{\partial^2 T_2(r, x, \tau)}{\partial x^2} = \frac{1}{a}\frac{\partial T_2(r, x, \tau)}{\partial \tau}$$

with the boundary conditions

$$T_1(r, x, 0) = T_2(r, x, 0) = T_0 = \text{const};$$

$$-\lambda\frac{\partial T_1(r, 0, \tau)}{\partial x} = q(\tau), \quad 0 \leqslant r \leqslant r_0; \quad \frac{\partial T_1(0, x, \tau)}{\partial r} = 0,$$

$$\frac{\partial T_2(r, 0, \tau)}{\partial x} = 0, \; r > r_0; \quad \frac{\partial T_1(r, \infty, \tau)}{\partial x} = \frac{\partial T_2(r, \infty, \tau)}{\partial x} = 0;$$

$$\frac{\partial T_2(\infty, x, \tau)}{\partial r} = 0; \quad T_1(r_0, x, \tau) = T_2(r_0, x, \tau);$$

$$\frac{\partial T_1(r_0, x, \tau)}{\partial r} = \frac{\partial T_2(r_0, x, \tau)}{\partial r}.$$

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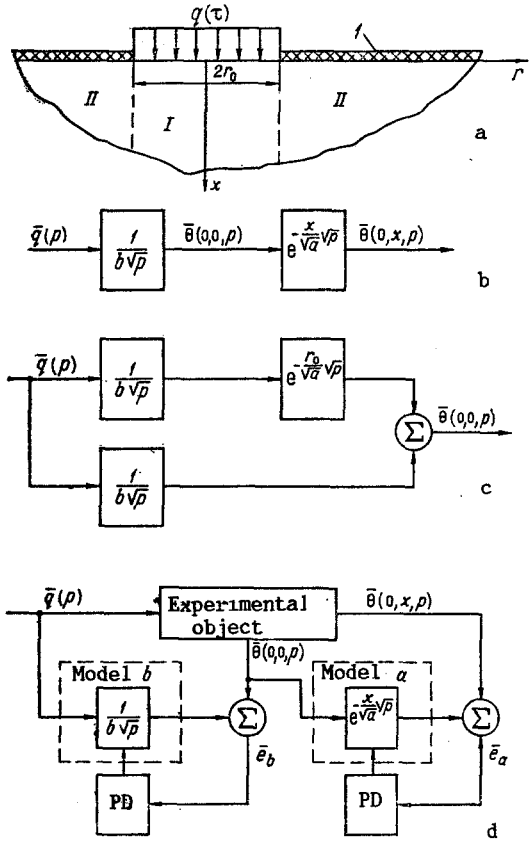

Fig. 1. Structural schemes for identifying the coefficients b and a of the thermal object: a) thermal model of the object with ideal insulation (1); b, c) formation of temperature fields inside the object and at its surface; d) identification of coefficients b and a.

The solution for the L transform of the excess temperature $\bar{\Theta}_1(0, x, p)$ [2]:

$$\bar{\Theta}_1(0, x, p) = \bar{q}(p)\left[\frac{1}{b\sqrt{p}}\exp\left(-\frac{x}{\sqrt{a}}\sqrt{p}\right) - \frac{1}{b\sqrt{p}}\exp\left(-\frac{\sqrt{r_0^2+x^2}}{\sqrt{a}}\sqrt{p}\right)\right].$$

As $r_0 \to \infty$, the expression is simplified

$$\bar{\Theta}_1(0, x, p) = \bar{q}(p)\left[\frac{1}{b\sqrt{p}}\exp\left(-\frac{x}{\sqrt{a}}\sqrt{p}\right)\right].$$

At the surface of the body (x = 0), the excess temperature is

$$\bar{\Theta}_1(0, 0, p) = \bar{q}(p)\left[\frac{1}{b\sqrt{p}} - \frac{1}{b\sqrt{p}}\exp\left(-\frac{r_0}{\sqrt{a}}\sqrt{p}\right)\right].$$

The structural schemes of temperature-field formation along the x axis $\bar{\Theta}(0, x, p)$ as $r_0 \to \infty$ and at the surface of the body $\bar{\Theta}(0, 0, p)$ ($r_0 \neq \infty$) are shown in Fig. 1b, c, respectively.

The mathematical model of temperature formation is represented by the operators $1/b\sqrt{p}$ and $\exp[-(x/\sqrt{a})\sqrt{p}]$ with the unknown coefficients b and a. These coefficients may be determined by calculation from the known values of the L transforms of the temperatures $\bar{\Theta}(0, 0, p)$ and $\bar{\Theta}(0, x, p)$ and the input perturbation $\bar{q}(p)$ [1]

$$b = \frac{1}{\sqrt{p}}\frac{\bar{q}(p)}{\bar{\Theta}(0, 0, p)}; \quad a = \frac{px^2}{\ln^2[\bar{\Theta}(0, x, p)/\bar{\Theta}(0, 0, p)]}.$$

In practice, temperature variation occurs over a time region with some error $\pm\delta$, and the L mapping includes an additional error due to the finite measurement interval and approximate calculation of the Laplace transformation. Therefore, the theoretical values of the coefficients b and a do not satisfy the requirement of desirable accuracy.

The coefficients b and a may be determined using an identification system based on models with tunable parameters, the controlling perturbations of which are the values of the errors e_b and e_a. The law of parameter tuning of the model is formed by program devices (PD) ensuring the minimization of the error [3]. The structural scheme of the identification system for coefficients b and a is shown in Fig. 1d.

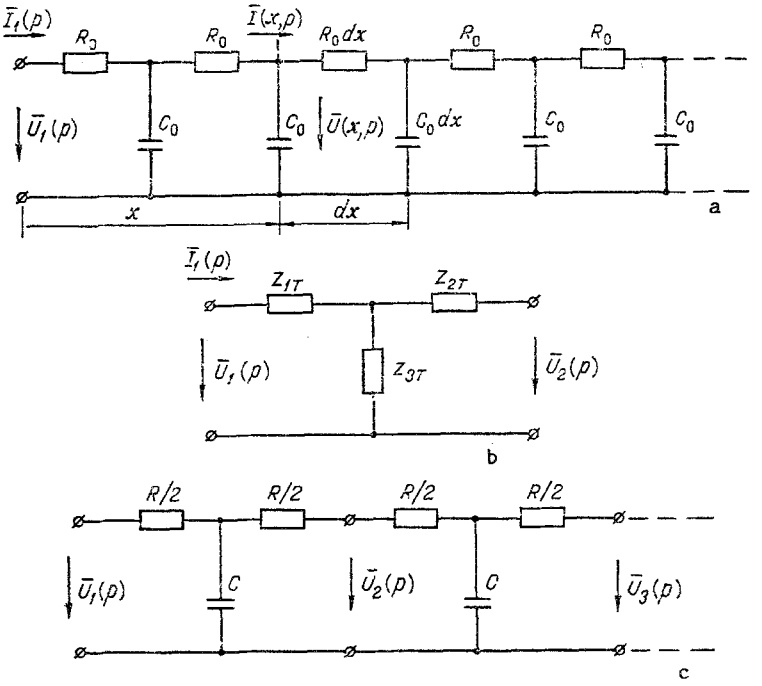

Fig. 2. Diagram of distributed RC structure and its discrete analog: a) RC structure; b) equivalent quadrupole; c) discrete analog of RC structure.

Models with tunable parameters b and a are described by the transfer functions $1/b\sqrt{p}$ and $\exp[-(x/\sqrt{a})\sqrt{p}]$, respectively, which are not expressed by ordinary differential equations. Therefore, they cannot be synthesized on the basis of electrical circuits with point parameters. The procedure for model synthesis is based on electrical circuits with distributed parameters, using the analogy of thermal and electrical processes occurring in a line with distributed parameters and described by a system of partial differential equations [4]

$$-\frac{\partial U(x,t)}{\partial x} = R_0 i(x,t) + L_0 \frac{\partial i(x,t)}{\partial t};$$
$$-\frac{\partial i(x,t)}{\partial x} = G_0 U(x,t) + C_0 \frac{\partial U(x,t)}{\partial t},$$

where R_0, G_0, L_0, C_0 are the line parameters.

Differentiating the first equation with respect to x and substituting the second into the result with $G_0 = L_0 = 0$, it is found that

$$\frac{\partial U(x,t)}{\partial t} = \frac{1}{R_0 C_0}\frac{\partial^2 U(x,t)}{\partial x^2}. \qquad (1)$$

Equation (1) is similar in form to the heat-conduction equation

$$\frac{\partial \Theta(x,\tau)}{\partial \tau} = a\frac{\partial^2 \Theta(x,\tau)}{\partial x^2}.$$

Differential Eq. (1) describes the space–time formation of the potential and current in a distributed RC structure (RC structure) characterized by a longitudinal resistance R_0 (Ω/m) and a linear capacity C_0 (F/m). A diagram of the RC structure is shown in Fig. 2a.

The potential and current in the cross section x, $\bar{U}(x, p)$ and $\bar{I}(x, p)$, are related to the potential and current at the beginning of the RC structure, $\bar{U}_1(p)$ and $\bar{I}_1(p)$, as follows [5]

$$\bar{U}(x,p) = \bar{U}_1(p)\operatorname{ch}\gamma x - \bar{I}_1(p) z_w \operatorname{sh}\gamma x;$$
$$\bar{I}(x,p) = \bar{I}_1(p)\operatorname{ch}\gamma x - \frac{\bar{U}_1(p)}{z_w}\operatorname{sh}\gamma x. \qquad (2)$$

TABLE 1. Table of Values of the Transition Functions $h_1(t)$ and $h_2(t)$

Coefficient k	Time t, sec	Transition function $h_2(t)$	Transfer function $h_1(t)$	Relative error δ, %	Coefficient k	Time t, sec	Transition function $h_2(t)$	Transfer function $h_1(t)$	Relative error δ, %
0,001	1	0,4767	0,4767	0,000	0,1	5	10,0774	10,0770	0,004
0,001	2	1,9337	1,9337	0,000	1,0	1	0,0967	0,0596	—
0,001	3	4,3779	4,3779	0,000	1,0	2	0,6447	0,6063	5,950
0,001	4	7,8117	7,8117	0,000	1,0	3	1,8044	1,7654	2,159
0,001	5	12,2365	12,2365	0,000	1,0	4	3,6452	3,6058	1,078
0,01	1	0,4296	0,4296	0,000	1,0	5	6,2086	6,1690	0,637
0,01	2	1,7969	1,7969	0,000	1,0	6	9,5232	9,4835	0,417
0,01	3	4,1237	4,1237	0,000	1,0	7	13,6104	13,5705	0,293
0,01	4	7,4178	7,4178	0,000	1,0	8	18,4865	18,4465	0,216
0,01	5	11,6835	11,6835	0,000	1,0	9	24,1650	24,1249	0,165
0,1	1	0,3066	0,3062	0,131	1,0	10	30,6571	30,6169	0,131
0,1	2	1,4191	1,4187	0,028	1,0	11	37,9721	37,9319	0,106
0,1	3	3,4040	3,4036	0,011	1,0	12	46,1182	46,0779	0,087
0,1	4	6,2854	6,2850	0,006					

For an $\overline{RC}$ structure of finite length ℓ, the relation between the input and output values is

$$\overline{U}_1(p) = \overline{U}_2(p)\operatorname{ch}\gamma l + \overline{I}_2(p) z_w \operatorname{sh}\gamma l;$$

$$\overline{I}_1(p) = \frac{\overline{U}_2(p)}{z_w}\operatorname{sh}\gamma l + \overline{I}_2(p)\operatorname{ch}\gamma l.$$

With a matched load $z_{1o} = z_w$, there are no reflected waves in the $\overline{RC}$ structure and its input resistance is equal to the wave resistance

$$z_{in} = \frac{\overline{U}_1(p)}{\overline{I}_1(p)} = \frac{z_{1o}\operatorname{ch}\gamma l + z_w \operatorname{sh}\gamma l}{z_{1o}/z_w \operatorname{sh}\gamma l + \operatorname{ch}\gamma l} = \sqrt{\frac{R_0}{pC_0}}.$$

The transfer function with respect to the potential of a matched $\overline{RC}$ structure of length ℓ is analogous to the transfer function of a thermal element with $x/\sqrt{a} = \sqrt{R_0 C_0}\,\ell$

$$W(p) = \frac{\overline{U}(x, p)}{\overline{U}_1(p)} = \operatorname{ch}\gamma l - \operatorname{sh}\gamma l = \exp(-\sqrt{pR_0C_0}\,l).$$

A distributed RC structure of length ℓ may be synthesized in a definite frequency region using circuits with point parameters described by ordinary differential equations. To this end, the analogy between the equations of the $\overline{RC}$ structure and the equations of a quadrupole in the A form is used, and the transfer matrix of a quadrupole is written

$$[A] = \begin{bmatrix} \operatorname{ch}\gamma l & z_w \operatorname{sh}\gamma l \\ \frac{1}{z_w}\operatorname{sh}\gamma l & \operatorname{ch}\gamma l \end{bmatrix} = \begin{bmatrix} \operatorname{ch}\sqrt{pRC} & z_w \operatorname{sh}\sqrt{pRC} \\ \frac{1}{z_w}\operatorname{sh}\sqrt{pRC} & \operatorname{ch}\sqrt{pRC} \end{bmatrix}.$$

With the aim of simplifying the realization, $\operatorname{sh}\sqrt{pRC}$ and $\operatorname{ch}\sqrt{pRC}$ are expanded in series and, limiting the expansions to the first terms, the approximate transfer A matrix of a quadrupole is written

$$[A]_{ap} = \begin{bmatrix} A + \frac{pRC}{2} & R \\ pC & 1 + \frac{pRC}{2} \end{bmatrix}.$$

When $RC \ll 1$, the relative error of the expansion is no greater than 0.1%; therefore, $[A] = [A]_{ap}$.

The transfer function of a matched quadrupole with the matrix [A] is

$$W_{ap}(p) = \frac{z_{1o}}{Az_{1o} + B} = \frac{1}{1 + \sqrt{pRC} + \frac{pRC}{2}} = \exp(-\sqrt{pRC})\,|_{RC \ll 1}.$$

The synthesis of a quadrupole with the matrix $[A]_{ap}$ is based on discrete components, using the symmetric replacement T scheme in Fig. 2b.

The parameters z_{1T} and z_{3T} are determined:

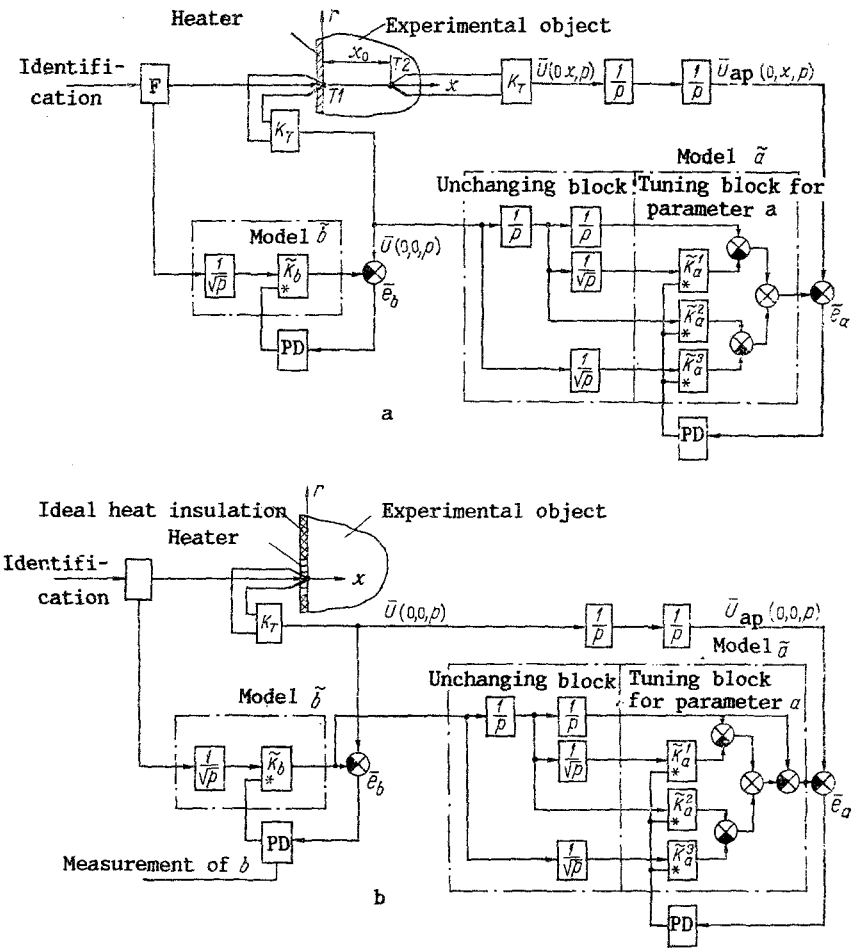

Fig. 3. Identification scheme for the coefficients a and b: a) with introduction into the experimental object; b) nondestructive monitoring.

$$z_{1T} = \frac{1 + \frac{pRC}{2} - 1}{pC} = \frac{R}{2};\ z_{3T} = \frac{1}{pC}.$$

The replacement scheme of the $\overline{RC}$ structure in the form of a cascade system of quadrupoles is shown in Fig. 2c.

The problem of synthesizing a model with a transfer function $\exp(-k\sqrt{p})$ and tunable parameter k based on the use of distributed RC structures is reduced to isolating the basic (unchanging) part of the model, ensuring the time formation of the output signal in the specified range of variation of k and the tuning block of the parameter k. To isolate the unchanging part of the model, the transfer function $\exp(-k\sqrt{p})$ is expanded in power series

$$\exp(-k\sqrt{p}) = 1 - k\sqrt{p} + \frac{(k\sqrt{p})^2}{2!} - \frac{(k\sqrt{p})^3}{3!} + \ldots = \sum_{m=0}^{\infty} (-1)^m \frac{(k\sqrt{p})^m}{m!}. \quad (3)$$

Retaining a sufficient number of terms in the expansion, a model with any degree of accuracy may be synthesized. Retaining four terms of the expansion, Eq. (3) takes the form

$$\frac{\exp(-k\sqrt{p})}{p^2} = \frac{1}{p^2} - \frac{k}{p\sqrt{p}} + \frac{k^2}{2p} - \frac{k^3}{6\sqrt{p}}. \quad (4)$$

Analysis of the error of the expansion in Eq. (4) is undertaken in the time region with a unit step perturbation at the input of the model. The relative error of the expansion is

$$\delta = \frac{|h_2(t) - h_1(t)|}{|h_2(t)|} 100\%,$$

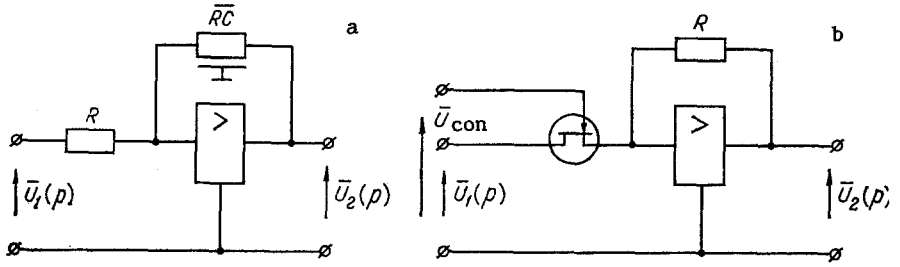

Fig. 4. Diagrams of elements: a) with the irrational transfer function $1/\sqrt{p}$; b) with a controllable transfer coefficient.

where

$$h_1(t) = \frac{t^2}{2} - \frac{4k}{3\sqrt{\pi}} t^{3/2} + \frac{k^2}{2} t - \frac{1}{3} \frac{k^{3/2}}{\sqrt{\pi}} t^{1/2},$$

$$h_2(t) = \operatorname{erfc} \frac{k}{2\sqrt{t}} \left[\frac{t^2}{2} + \frac{k^2}{2} t + \frac{k^4}{24} \right] - \frac{k e^{-k^2/4t}}{\sqrt{\pi}} \left[\frac{5}{6} t^{3/2} + \frac{k^2}{12} t^{1/2} \right].$$

Calculations of the relative error of the expansion in Eq. (4) with various values of k are performed on a computer, with the results in Table 1. Analysis of the calculations shows that, to ensure an accuracy of 0.1%, the range of variation in k must be within the limits $0 < k \leq 1$ ($t > 10$ sec).

A structural diagram of the model described by the series in Eq. (4) is shown in Fig. 3a (model ã). The unchanging part of the model is based on typical integrating elements with the transfer function $1/p$ and special semiintegrating elements with transfer function $1/\sqrt{p}$. The element with the irrational transfer function $1/\sqrt{p}$ is described by partial differential equations and therefore is constructed using a distributed RC structure. The scheme of the semiintegrating element based on an ideal operational amplifier with a matched $\overline{RC}$ structure in the feedback circuit is shown in Fig. 4a. The transfer function of the element is [6]

$$W(p) = \frac{z_w}{R} = \frac{\sqrt{R_0/pC_0}}{R} = \frac{\alpha}{\sqrt{p}}, \quad \alpha = \sqrt{\frac{R_0}{R^2 C_0}}.$$

The element with transfer function $1/p$ was described in [3]. The changing part of the model (tuning block for the parameter k) is based on noninertial elements with a controllable transfer function $\tilde{K}$ and summation and subtraction elements. The scheme of the noninertial element based on an ideal operative amplifier with a field transistor at the input is shown in Fig. 4b. The transfer function of the element is

$$K(p) = \frac{R}{R_{ss}} = \tilde{K},$$

where R_{ss} is the sink–source resistance of the field transistor. The summation and subtraction elements were described in [7].

The identification scheme for the coefficients a and b with the introduction into the experimental object is shown in Fig. 3a. Step functions of the heat flux fed to the experimental object and the potential are formed on the basis of the identification signal by the unit forming the input perturbations (F). The signal from thermocouple T1 is amplified and compared with the output signal of model $\tilde{b}$. In accordance with the error e_b, the program device develops a controlling perturbation at element $\tilde{K}_b$. The tuning algorithm for $\tilde{K}_b$ ensuring minimization of the error was described in [8]. The signal from thermocouple T2 is transformed (filtered) and compared with the output signal from model ã. To ensure the required accuracy, the range of variation must be within the limits $0 < \tilde{K}_a \leq 1$; therefore, x_0 is chosen from the condition $x_0/\sqrt{a} \leq 1$. The numerical value of the coefficient a for most nonmetallic materials is in the range $(0.5\text{-}50)\cdot 10^{-7}$; therefore, for these materials, $x_0 \leq (0.2\text{-}2)\cdot 10^{-3}$ m.

A diagram of the nondestructive method of identifying the coefficients a and b (at the surface of the given material) is shown in Fig. 3b. The identification process has two stages. In the first stage, the coefficient b is found; then, with known b, a is identified. The time to identify the coefficient b is determined from the condition $t_b \leq r_0^2 \cdot 10^3$, sec.

NOTATION

$T(r, x, \tau)$, temperature at any point of the experimental object; r, x, current coordinates of object; τ, time; $\bar{\theta}(r, x, \tau) = T(r, x, \tau) - T_0$, excess temperature of experimental object; $q(\tau)$, heat-flux density; $U(x, t)$, potential in cross section x of long line; $i(x, t)$, current in cross section x of long line; x, current coordinate of long line; t, time; γ, constant of propagation; p, Laplace-transformation parameter; z_w, wave resistance of RC structure; z_{lo}, load resistance; A, B, matrix elements of quadrupole; $h_1(t)$, $h_2(t)$, transition characteristics of the models; k, tunable coefficient; erfc x = 1 − erf x; erf $x = 2/\sqrt{\pi} \int_0^x e^{-x^2} dx$, Gaussian error function.

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FORMULAS FOR THE DISCREPANCY GRADIENT IN THE ITERATIVE SOLUTION OF INVERSE HEAT-CONDUCTION PROBLEMS. II. DETERMINING THE GRADIENT IN TERMS OF A CONJUGATE VARIABLE

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The construction of the functional-deficiency gradient is considered for the iterative solution of inverse problems in the case of an equation of parabolic type. Nonlinear formulations of the problem are considered in the general case.

In the first part of this report [1], formulas were obtained for the discrepancy gradient in terms of the Green's function of the corresponding boundary problem. A more general method of finding the gradient is based on solving the conjugate boundary problem [2, 3]. Below, an approach to deriving the conditions of this problem and formulas for the discrepancy gradient allowing a rigorous basis for the results obtained to be established is outlined.

Suppose that in a region with mobile boundaries $Q_\tau = \{X_1(\tau) < x < X_2(\tau),\ 0 < \tau < \tau_m\}$ a quasilinear parabolic equation is specified

$$C T_\tau = (\lambda T_x)_x + K T_x + g. \tag{1}$$

The initial and boundary conditions for Eq. (1) are

$$T(x, 0) = \xi(x), \tag{2}$$

$$[\alpha_i \lambda T_x + \beta_i T]_{x = X_i(\tau)} = p_i(\tau), \quad i = 1, 2, \tag{3}$$

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